Generalized Ratio Type Estimator of Population Mean Under Ranked Set Sampling

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Abstract

In this paper, a ratio type estimator for population mean based on simple random sampling (SRS) and ranked set sampling (RSS) has been proposed. The expressions for mean square errors and its minimum values have been obtained. The proposed estimator under SRS design has been shown to be more efficient than the existing estimators. It has also been shown that proposed estimator under RSS design is always better than that under SRS design. Further, the proposed estimator has been shown to be better than the estimators proposed by Jemain et al [1] and Al-Omari et al [2]. A simulation study is performed to verify results of the proposed estimators.

Keywords: Efficiency, Ranked Set Sampling, Ratio Estimator, Simple Random Sampling.

1. INTRODUCTION

Ranked set sampling was suggested by McIntyre [5] to estimate population mean as a more efficient and cost effective method than the commonly used simple random sampling in the situations when the measure of study variable(y) is difficult and expensive but ordering of sampled units can be done easily by using any inexpensive method. Takahasi and Wakimoto [8] provided the necessary mathematical theory of RSS. Samawi and Muttlak [6] suggested ratio estimator under RSS to estimate population ratio that can also be used for population mean and population total. Jemain et al [1] suggested a modified ratio estimator for population mean using RSS and it has been found that the modified ratio estimator for population mean using RSS is more efficient than the estimator proposed by Singh and Tailor [7].

In the present paper, SRS and RSS designs are used to propose the ratio type estimators for estimating the population mean of the study variable(y), using known information on the auxiliary variable(x). The expressions for mean square errors and its minimum values have
been obtained. This estimator for population mean under RSS is also compared with the estimator suggested by Jemain et al [1] and Al-Omari et al [2].

1.1. RSS METHOD

The RSS procedure for selecting a sample of size $n'$, involves selecting randomly $n'^2$ units from the population and allocating randomly into $n'$ equal samples. The $n'$ units of each sample are ranked visually or by any inexpensive method with respect to the study variable. From the first set of $n'$ units, the smallest unit is taken and from the second set, the second smallest of $n'$ units is taken. The process is continued until upto the $n^{th}$ set, from where the largest of the $n'$ units is measured. Repeating the process $m$ times yields a sample of size $mn' = n$ from $mn'^2$ units. Due to these $m$ repetitions, the MSE of the estimators decreases $m$ times.

2. UNDER SRS DESIGN

Sisodia and Dwivedi [4], Singh and Tailor [7] and Al-Omari et al [2] proposed the ratio type estimators of population mean ($\bar{Y}$) by using the known information of population parameters.

General form of these estimators is given by

$$\bar{y}_{R\beta} = \bar{y}
\left(\frac{\overline{X} + \beta}{x + \beta}\right),$$  

(1)

where $\beta$ is the known value of a population parameter; $\bar{y}, \bar{x}$ are the sample means of study variable and auxiliary variable respectively and $\overline{X}$ denotes the population mean of auxiliary variable.

One can easily obtain different estimators of population mean ($\bar{Y}$) by choosing different values of $\beta$. For example

1. When $\beta = C_x$, where $C_x$ is the coefficient of variation of auxiliary variable(x), $\bar{y}_{R\beta}$ reduces to $\bar{y}_{sp} = \bar{y}
\left(\frac{\overline{X} + C_x}{x + C_x}\right)$ which was defined by Sisodia and Dwivedi(1981).
2. When $\beta = \rho$, where $\rho$ is the coefficient of correlation between auxiliary variable(x) and study variable(y), then $\bar{y}_{R\beta}$ reduces to $\bar{y}_{RST} = \left(\frac{\bar{X} + \rho}{\bar{x} + \rho}\right)$ which was proposed by Singh and Tailor [7].

3. When $\beta = q_1$ or $q_3$ where $q_1$ and $q_3$ are first and third quartiles of auxiliary variable(x), then $\bar{y}_{R\beta}$ reduces to $\bar{y}_{AO1} = \left(\frac{\bar{X} + q_1}{\bar{x} + q_1}\right)$ or $\bar{y}_{AO3} = \left(\frac{\bar{X} + q_3}{\bar{x} + q_3}\right)$ respectively, which were defined by Al-Omari et al [2].

The bias and MSE of $\bar{y}_{R\beta}$ are

$$bias(\bar{y}_{R\beta}) = \left(\frac{1}{n} - \frac{1}{N}\right)[K_{\beta}\theta_{\beta}S_x^2 - \theta_{\beta}S_{xy}]$$  \hspace{1cm} (2)

and

$$MSE(\bar{y}_{R\beta}) = \left(\frac{1}{n} - \frac{1}{N}\right)[S_y^2 + K_{\beta}^2S_x^2 - 2K_{\beta}S_{yx}]$$  \hspace{1cm} (3)

where $K_{\beta} = \frac{\bar{Y}}{\bar{X} + \beta}$ and $\theta_{\beta} = \frac{1}{\bar{X} + \beta}$.

For $N \gg n$

$$bias(\bar{y}_{R\beta}) = \frac{1}{n}[K_{\beta}\theta_{\beta}S_x^2 - \theta_{\beta}S_{xy}]$$  \hspace{1cm} (4)

and

$$MSE(\bar{y}_{R\beta}) = \frac{1}{n}[S_y^2 + K_{\beta}^2S_x^2 - 2K_{\beta}S_{yx}]$$  \hspace{1cm} (5)

Where $S_x^2$ and $S_y^2$ denote the population variance of auxiliary variable and study variable respectively and $S_{yx}$ is the population covariance between auxiliary variable and study variable [ see Cochran [11]].

A ratio type estimator of population mean ($\bar{Y}$) is proposed by using the power transformation as:

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\[- \bar{y}_{BM} = y \left( \frac{x + \beta}{X + \beta} \right)^{\alpha}, \quad (6)\]

Where \( \alpha \) and \( \beta \) are known constants. One can obtain the different estimators by choosing different values of \( \alpha \) and \( \beta \). For example,

1. When \( \alpha = 0 \); \( \bar{y}_{BM} \) becomes mean per unit estimator.
2. When \( \alpha = 1, \beta = 0 \); \( \bar{y}_{BM} \) becomes classical product estimator.
3. When \( \alpha = -1, \beta = 0 \); \( \bar{y}_{BM} \) becomes classical ratio estimator.
4. When \( \beta = 0 \); \( \bar{y}_{BM} \) becomes power transformation estimator [see Srivastava(1967)].

The bias and MSE of \( \bar{y}_{BM} \) are given by

\[
\text{bias}(\bar{y}_{BM}) = \left( \frac{1}{n} - \frac{1}{N} \right) \left[ \frac{\alpha(\alpha-1)}{2} K_{\beta} \theta_{\beta} S^2_x + \alpha \theta_{\beta} S_{xy} \right], \quad (7)
\]

and

\[
\text{MSE}(\bar{y}_{BM}) = \left( \frac{1}{n} - \frac{1}{N} \right) \left[ S^2_y + \alpha^2 K_{\beta}^2 S^2_x + 2 \alpha K_{\beta} S_{yx} \right], \quad (8)
\]

For \( N \gg n \), we have

\[
\text{bias}(\bar{y}_{BM}) = \frac{1}{n} \left[ \frac{\alpha(\alpha-1)}{2} K_{\beta} \theta_{\beta} S^2_x + \alpha \theta_{\beta} S_{xy} \right], \quad (9)
\]

and

\[
\text{MSE}(\bar{y}_{BM}) = \frac{1}{n} \left[ S^2_y + \alpha^2 K_{\beta}^2 S^2_x + 2 \alpha K_{\beta} S_{yx} \right] \quad (10)
\]

The value of which minimizes \( \text{MSE}(\bar{y}_{BM}) \) can be obtained after solving the following equation:

\[
\frac{\partial \text{MSE}(\bar{y}_{BM})}{\partial \alpha} = 0.
\]

The optimum value of \( \alpha \) is
\[
\alpha_{opt} = -\rho \frac{S_y}{S_x} \frac{\bar{X} + \beta}{\bar{Y}} = -\rho \frac{S_y}{K_\rho S_x} = \alpha' \text{(say)}. \tag{11}
\]

Substituting the value of \( \alpha = \alpha' \) in (8), we obtain minimum \( MSE(\bar{y}_{BM}) \) as

\[
MSE_{\min}(\bar{y}_{BM}) = \left( \frac{1}{n} - \frac{1}{N} \right) (1 - \rho^2) S_y^2. \tag{12}
\]

From equations (3) and (12), we get

\[
MSE(\bar{y}_{RR}) - MSE_{\min}(\bar{y}_{BM}) = \left( \frac{1}{n} - \frac{1}{N} \right) \left( \rho S_y - K_\rho S_x \right)^2 \geq 0 \tag{13}
\]

Above equation shows that the proposed estimator is more efficient than the estimators defined by Sisodia and Dwivedi [4], Singh and Tailor [7] and Al-Omari et al [2].

The optimum value of \( \alpha \) i.e. \( \alpha' \) depends upon the values of unknown parameters. Therefore, the estimator cannot work for practical problems. Following corollary addresses this problem.

**COROLLARY 1.** The modified estimator of \( \bar{y}_{BM} \) may be used practically which can be obtained by replacing \( \alpha \) by a consistent estimator of \( \alpha' \) in (6) as given below

\[
\bar{y}_{BM} = \bar{y} \left( \frac{\bar{x} + \beta}{X + \beta} \right)^{\hat{\alpha}'}
\]

where \( \hat{\alpha}' \) is a consistent estimator of \( \alpha' \). For example, \( \hat{\alpha}' = -\rho \frac{s_y}{s_x} \frac{\bar{X} + \beta}{\bar{Y}} \), where \( s_y \) and \( s_x \) are sample standard deviations of study and auxiliary variables respectively.

### 3. UNDER RSS DESIGN

Let \( \{ y_{j(i)}, x_{j(i)} \} \) denote an \( i^{th} \) unit in the \( j^{th} \) ranked set sample, where \( i = 1, 2, ..., n' ; j = 1, 2, ..., m \) and \( mm' = n \). Jemain et al [1] and Al-Omari et al [2] defined the ratio estimators of population mean(\( \bar{Y} \)) by using RSS. General form of these estimators is defined as

\[
\bar{y}_{R_{RSS}} = \bar{y}_{RSS} \left( \frac{\bar{X} + \beta}{x_{RSS} + \beta} \right). \tag{14}
\]
where $\beta$ is a pre-assigned constant, \( \bar{y}_{rss} = \frac{1}{mn} \sum_{j=1}^{m} \sum_{i=1}^{n} y_{j(i)} \) and \( \bar{x}_{rss} = \frac{1}{mn} \sum_{j=1}^{m} \sum_{i=1}^{n} x_{j(i)} \).

By choosing different values of $\beta$, one can obtain different estimators of population mean as follow

1. When $\beta = C_x$, $\bar{y}_{RSS} \longrightarrow \bar{y}_{SDRSS} = \frac{\bar{X} + C_x}{x_{RSS} + C_x}.$

2. When $\beta = \rho$, $\bar{y}_{RSS} \longrightarrow \bar{y}_{RSS} = \frac{\bar{X} + \rho}{x_{RSS} + \rho}$ which was proposed by Jemain et al [1].

3. When $\beta = q_1$ or $q_3$, $\bar{y}_{RSS} \longrightarrow \bar{y}_{AOHSS} = \frac{\bar{X} + q_1}{x_{RSS} + q_1}$ or $\bar{y}_{AOHSS} = \frac{\bar{X} + q_3}{x_{RSS} + q_3}$ respectively which were defined by Al-Omari et al [2].

The bias and MSE of this estimator can be given by

\[
\text{bias}(\bar{y}_{RSS}) = K_{\beta} \theta, \text{V}(\bar{x}_{RSS}) - \theta, \text{Cov}(x_{RSS}, \bar{y}_{RSS})
\]

and

\[
\text{MSE}(\bar{y}_{RSS}) = \text{V}(\bar{y}_{RSS}) + K_{\beta}^2 \text{V}(\bar{x}_{RSS}) - 2K_{\beta} \text{Cov}(\bar{x}_{RSS}, \bar{y}_{RSS}),
\]

where

\[
\text{V}(\bar{y}_{RSS}) = \frac{1}{mn} \left( S_y^2 - \frac{1}{n} \sum_{i=1}^{n} r_{y(i)}^2 \right),
\]

(17)

\[
\text{V}(\bar{x}_{RSS}) = \frac{1}{mn} \left( S_x^2 - \frac{1}{n} \sum_{i=1}^{n} r_{x(i)}^2 \right),
\]

(18)

and

\[
\text{Cov}(\bar{x}_{RSS}, \bar{y}_{RSS}) = \frac{1}{mn} \left( S_{yx} - \frac{1}{n} \sum_{i=1}^{n} r_{y(i)} \right),
\]

(19)
Here \( \tau_{x(i)} = \mu_{x(i)} - \bar{X} \), \( \tau_{y(i)} = \mu_{y(i)} - \bar{Y} \) and \( \tau_{y(s)} = \left( \mu_{y(i)} - \bar{Y} \right) \left( \mu_{x(i)} - \bar{X} \right) \). Note that the values of \( \mu_{x(i)} \) and \( \mu_{y(i)} \) depend on order statistics from some specific distributions and these values can be found in Arnold et al. [3]. We would like to point out that the values of \( \mu_{x(i)} \) and \( \mu_{y(i)} \) can be taken to be same in the absence of judgment errors if the variables have the same distribution [see the appendix of Dell and Clutter [10]].

Using (17), (18), and (19) in (16), we get

\[
MSE(\bar{y}_{\text{RSS}}) = \frac{1}{mn} \left[ S_y^2 + K_\beta^2 S_x^2 - 2 K_\beta S_{yx} \right] - \frac{1}{mn^2} \sum_{i=1}^{n} \left[ \tau_{y(i)} - K_\beta \tau_{x(i)} \right]^2.
\]  

(20)

From (3) and (20), we have

\[
MSE(\bar{y}_\beta) - MSE(\bar{y}_{\text{RSS}}) = \frac{1}{mn^2} \sum_{i=1}^{n} \left[ \tau_{y(i)} - K_\beta \tau_{x(i)} \right]^2 \geq 0,
\]  

(21)

which shows that \( \bar{y}_{\text{RSS}} \) is always more efficient than \( \bar{y}_\beta \).

A ratio type estimator of population mean (\( \bar{Y} \)) under RSS design is proposed as

\[
\bar{y}_{BM\text{RSS}} = y_{\text{RSS}} \left( \frac{x_{\text{RSS}} + \beta}{X + \beta} \right)^\alpha
\]  

(22)

where \( \alpha \) and \( \beta \) are pre-assigned constants.

The bias and MSE of the estimator are given by

\[
bias(\bar{y}_{BM\text{RSS}}) = \frac{\alpha(\alpha - 1)}{2} K_\beta \theta_\beta V(x_{\text{RSS}}) + \alpha \theta_\beta Cov(x_{\text{RSS}}, \bar{y}_{\text{RSS}})
\]  

(23)

\[
= \frac{1}{mn} \left[ \frac{\alpha(\alpha - 1)}{2} K_\beta \theta_\beta S_x^2 + \alpha \theta_\beta S_{yx} \right] - \frac{1}{mn^2} \left[ \frac{\alpha(\alpha - 1)}{2} K_\beta \theta_\beta \sum_{i=1}^{n} \tau_{x(i)}^2 + \alpha \theta_\beta \sum_{i=1}^{n} \tau_{y(s)} \right]
\]

and

\[
MSE(\bar{y}_{BM\text{RSS}}) = V(\bar{y}_{\text{RSS}}) + \alpha^2 K_\beta^2 V(x_{\text{RSS}}) + 2 \alpha K_\beta Cov(x_{\text{RSS}}, \bar{y}_{\text{RSS}})
\]  

(24)

\[
= \frac{1}{mn} \left[ S_y^2 + \alpha^2 K_\beta^2 S_x^2 + 2 \alpha K_\beta S_{yx} \right] - \frac{1}{mn^2} \sum_{i=1}^{n} \left[ \tau_{y(i)} + \alpha K_\beta \tau_{x(i)} \right]^2.
\]  

(25)
From (10) and (25), we get

\[
MSE(\tilde{y}_{BM}) - MSE(\tilde{y}_{BM_{rss}}) = \frac{1}{nn^2} \sum_{i=1}^{n} \left[ x_{y[i]} + \alpha K_{\beta} \tau_{s(i)} \right]^2 \geq 0,
\]

which shows that \( \tilde{y}_{BM_{rss}} \) is always more efficient than \( \tilde{y}_{BM} \) corresponding to each value of \( \alpha \).

The optimum value of \( \alpha \) which minimizes MSE of \( \tilde{y}_{BM_{rss}} \) can easily be obtained after solving the following equation:

\[
\frac{\partial MSE(\tilde{y}_{BM_{rss}})}{\partial \alpha} = 0,
\]

and that value of \( \alpha \) is

\[
\alpha_{opt} = -\frac{Cov(\bar{x}_{rss}, \bar{y}_{rss})}{V(\bar{x}_{rss})} \bar{X} + \beta \frac{\bar{Y}}{Y} = -\left( \frac{S_{xy} - \frac{1}{n} \sum_{i=1}^{n} \tau_{y[i]} \tau_{s[i]}}{S_{x}^2 - \frac{1}{n} \sum_{i=1}^{n} \tau_{s(i)}^2} \right) \bar{X} + \beta \frac{\bar{Y}}{Y} = \alpha^* \text{ (say)}
\]

After substituting \( \alpha = \alpha^* \) in (24), one can obtain the expressions for minimum MSE of \( \tilde{y}_{BM_{rss}} \) as follows:

\[
MSE_{\min}(\tilde{y}_{BM_{rss}}) = \left( 1 - \rho_{\bar{x}_{rss}, \bar{y}_{rss}}^2 \right) MSE(\bar{y}_{rss})
\]

where \( \rho_{\bar{x}_{rss}, \bar{y}_{rss}} \) is the correlation coefficient between \( \bar{x}_{rss} \) and \( \bar{y}_{rss} \).

The optimum value of \( \alpha \) (i.e. \( \alpha^* \)) depends upon the values of unknown parameters. So it cannot be used practically. Thus we have following corollary.

**COROLLARY 2.** The modified estimator of \( \tilde{y}_{BM_{rss}} \) may be used practically which can be obtained by replacing \( \alpha \) by a consistent estimator of \( \alpha^* \) in (22) as given below

\[
\tilde{y}_{BM_{rss (pract)}} = \bar{y}_{rss} \left( \frac{\bar{x}_{rss} + \beta}{\bar{X} + \beta} \right)^{\hat{\alpha}}
\]
where $\hat{\alpha}^*$ is a consistent estimator of $\alpha^*$. For example $\hat{\alpha}^* = \frac{\sum_{i=1}^{m} s_{x(i)} y}{\sum_{i=1}^{m} s_{x(i)}^2} \bar{X} + \beta$, where $s_{x(i)} = \frac{1}{m-1} \sum_{j=1}^{m} (x_{j(i)} - \bar{x}(i))(y_{j(i)} - \bar{y}(i))$ and $s_{x(i)}^2 = \frac{1}{m-1} \sum_{j=1}^{m} (x_{j(i)} - \bar{x}(i))^2$. Here $\bar{x}(i)$ and $\bar{y}(i)$ denote the sample means for $i^{th}$ order statistics of auxiliary and study variables respectively.

4. COMPARISON

**Theorem 1.** $\bar{y}_{BM_{rss}}$ is always more efficient than $\bar{y}_{J_{rss}}$.

**Proof:** From equations (16) and (28), we get

$$MSE(\bar{y}_{R_{rss}}) - MSE_{\min}(\bar{y}_{BM_{rss}}) = \left[ \rho_{x_{\alpha},y_{\alpha}} - \sqrt{V(\bar{y}_{rss})} - K_{\alpha} \sqrt{V(\bar{x}_{rss})} \right]^2 \geq 0$$

which shows that $\bar{y}_{BM_{rss}}$ is more efficient than $\bar{y}_{J_{rss}}$ at optimum value of $\alpha$.

**Theorem 2.** $\bar{y}_{BM_{rss}}$ is always more efficient than $\bar{y}_{BM}$.

**Proof:** From equation (26), at point $\alpha'$, we get

$$MSE_{\min}(\bar{y}_{BM}) \geq MSE_{\alpha'}(\bar{y}_{BM_{rss}}).$$

Since $\alpha'$ is an optimum point for $MSE(\bar{y}_{BM_{rss}})$, this implies

$$MSE_{\alpha'}(\bar{y}_{BM_{rss}}) \geq MSE_{\min}(\bar{y}_{BM_{rss}}).$$

From above two inequalities, we get

$$MSE_{\min}(\bar{y}_{BM}) \geq MSE_{\min}(\bar{y}_{BM_{rss}})$$

which shows that $\bar{y}_{BM_{rss}}$ is more efficient than $\bar{y}_{BM}$ corresponding to their optimum values of $\alpha$.

5. SIMULATION STUDY

A simulation study is conducted by using software R to study the properties of the estimators. 1,00,000 samples are generated from bivariate normal distribution $BVN(100,200,4,4,\rho)$ where $\rho = 0.3, 0.5, 0.7, 0.9$ under SRS design and RSS design.
The efficiency of an estimator \( \hat{\theta} \) with respect to \( \bar{y} \) to estimate population mean \( \bar{Y} \) is defined as:

\[
eff(\hat{\theta}) = \frac{V(\bar{y})}{\text{MSE}(\hat{\theta})}.
\]

The values of biases of the estimators for sample sizes 9, 12 and 15 are obtained in Table-1, Table-2 and Table-3 respectively and efficiencies of the estimators for sample sizes 9, 12 and 15 are obtained in Table-4, Table-5 and Table-6 respectively.

**Table-1: Biases of the estimators for \( n=9, n'=3 \) and \( m=3 \).**

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>bias(( \bar{y} ))</td>
<td>0.00080</td>
<td>0.00080</td>
<td>0.00080</td>
<td>0.00080</td>
</tr>
<tr>
<td>bias(( \bar{y}_{SD} ))</td>
<td>-0.00015</td>
<td>-0.00056</td>
<td>-0.00088</td>
<td>-0.00103</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{\rho} ))</td>
<td>0.00009</td>
<td>-0.00056</td>
<td>-0.00124</td>
<td>-0.00170</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{\sigma} ))</td>
<td>-0.00015</td>
<td>-0.00056</td>
<td>-0.00088</td>
<td>-0.00103</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{C_B} ))</td>
<td>0.00009</td>
<td>-0.00056</td>
<td>-0.00124</td>
<td>-0.00170</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AOI} ))</td>
<td>0.00004</td>
<td>-0.00016</td>
<td>-0.00032</td>
<td>-0.00040</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
<td>-0.00007</td>
<td>-0.00084</td>
<td>-0.00163</td>
<td>-0.00220</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
<td>0.00005</td>
<td>-0.00060</td>
<td>-0.00072</td>
<td>-0.00090</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
<td>-0.00007</td>
<td>-0.00084</td>
<td>-0.00163</td>
<td>-0.00220</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
<td>-0.00032</td>
<td>-0.00054</td>
<td>-0.00073</td>
<td>-0.00080</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
<td>-0.00009</td>
<td>-0.00036</td>
<td>-0.00070</td>
<td>-0.00105</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
<td>-0.00032</td>
<td>-0.00054</td>
<td>-0.00073</td>
<td>-0.00080</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
<td>-0.00009</td>
<td>-0.00036</td>
<td>-0.00071</td>
<td>-0.00105</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
<td>-0.00033</td>
<td>-0.00042</td>
<td>-0.00048</td>
<td>-0.00048</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
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<td>-0.00131</td>
</tr>
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<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
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<td>-0.00042</td>
<td>-0.00048</td>
<td>-0.00047</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM}^{AO3} ))</td>
<td>-0.00018</td>
<td>-0.00051</td>
<td>-0.00091</td>
<td>-0.00131</td>
</tr>
</tbody>
</table>
Table-2: Biases of the estimators for \( n=12, n'=4 \) and \( m=3 \).

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00126</td>
<td>0.00126</td>
<td>0.00126</td>
<td>0.00126</td>
</tr>
<tr>
<td>( \hat{\mu}_{AO} )</td>
<td>0.00107</td>
<td>0.00064</td>
<td>0.00024</td>
<td>-0.00013</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00105</td>
<td>0.00064</td>
<td>0.00006</td>
<td>-0.00064</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00107</td>
<td>0.00064</td>
<td>0.00006</td>
<td>-0.00012</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00105</td>
<td>0.00064</td>
<td>0.00006</td>
<td>-0.00064</td>
</tr>
<tr>
<td>( \hat{\mu}_{AO} )</td>
<td>0.00096</td>
<td>0.00074</td>
<td>0.00054</td>
<td>0.00036</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00093</td>
<td>0.00044</td>
<td>-0.00023</td>
<td>-0.00101</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00096</td>
<td>0.00075</td>
<td>0.00054</td>
<td>0.00036</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00092</td>
<td>0.00044</td>
<td>-0.00023</td>
<td>-0.00101</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00056</td>
<td>0.00031</td>
<td>0.00011</td>
<td>0.00004</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00116</td>
<td>0.00082</td>
<td>0.00030</td>
<td>-0.00045</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00056</td>
<td>0.00031</td>
<td>0.00011</td>
<td>0.00005</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00116</td>
<td>0.00082</td>
<td>0.00030</td>
<td>-0.00045</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00083</td>
<td>0.00072</td>
<td>0.00065</td>
<td>0.00065</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00111</td>
<td>0.00073</td>
<td>0.00018</td>
<td>-0.00061</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00083</td>
<td>0.00072</td>
<td>0.00065</td>
<td>0.00066</td>
</tr>
<tr>
<td>( \hat{\mu}_{BM} )</td>
<td>0.00111</td>
<td>0.00073</td>
<td>0.00018</td>
<td>-0.00061</td>
</tr>
</tbody>
</table>

Some results based on Table-1 to Table-6 are as follows:

1. \( \bar{y}_{BM} \) is always more efficient than the other estimators.

2. Efficiencies of all ratio-type estimators increase as the value of \( \rho \) increases.

3. For this particular population, all the estimators are approximately unbiased estimators of population mean.
Table-3: Biases of the estimators for \( n=15, n'=5 \) and \( m=3 \).

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>bias(( \bar{y} ))</td>
<td>0.00138</td>
<td>0.00138</td>
<td>0.00138</td>
<td>0.00138</td>
</tr>
<tr>
<td>bias(( \bar{y}_{SD} ))</td>
<td>0.00183</td>
<td>0.00139</td>
<td>0.00092</td>
<td>0.00040</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM} ))</td>
<td>0.00157</td>
<td>0.00139</td>
<td>0.00092</td>
<td>0.00010</td>
</tr>
<tr>
<td>bias(( \bar{y}_{AT} ))</td>
<td>0.00183</td>
<td>0.00139</td>
<td>0.00092</td>
<td>0.00040</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM} ))</td>
<td>0.00157</td>
<td>0.00139</td>
<td>0.00092</td>
<td>0.00010</td>
</tr>
<tr>
<td>bias(( \bar{y}_{AO} ))</td>
<td>0.00144</td>
<td>0.00121</td>
<td>0.00098</td>
<td>0.00072</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM} ))</td>
<td>0.00147</td>
<td>0.00122</td>
<td>0.00069</td>
<td>0.00020</td>
</tr>
<tr>
<td>bias(( \bar{y}_{AO} ))</td>
<td>0.00143</td>
<td>0.00121</td>
<td>0.00098</td>
<td>0.00072</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM} ))</td>
<td>0.00147</td>
<td>0.00122</td>
<td>0.00069</td>
<td>0.00020</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM} ))</td>
<td>0.00071</td>
<td>0.00074</td>
<td>0.00069</td>
<td>0.00039</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM} ))</td>
<td>0.00071</td>
<td>0.00074</td>
<td>0.00069</td>
<td>0.00039</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM} ))</td>
<td>0.00082</td>
<td>0.00071</td>
<td>0.00057</td>
<td>0.00041</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM} ))</td>
<td>0.00067</td>
<td>0.00068</td>
<td>0.00061</td>
<td>0.00029</td>
</tr>
<tr>
<td>bias(( \bar{y}_{BM} ))</td>
<td>0.00067</td>
<td>0.00068</td>
<td>0.00061</td>
<td>0.00029</td>
</tr>
</tbody>
</table>

Table-4: Efficiencies of the estimators for \( n=9, n'=3 \) and \( m=3 \).

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>eff(( \bar{y} ))</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>eff(( \bar{y}_{SD} ))</td>
<td>1.053248</td>
<td>1.334118</td>
<td>1.819217</td>
<td>2.858433</td>
</tr>
<tr>
<td>eff(( \bar{y}_{AT} ))</td>
<td>1.053569</td>
<td>1.334115</td>
<td>1.816935</td>
<td>2.843999</td>
</tr>
<tr>
<td>eff(( \bar{y}_{BM} ))</td>
<td>1.09644</td>
<td>1.231903</td>
<td>1.40552</td>
<td>1.636005</td>
</tr>
<tr>
<td>eff(( \bar{y}_{BM} ))</td>
<td>1.096235</td>
<td>1.230622</td>
<td>1.402527</td>
<td>1.630158</td>
</tr>
<tr>
<td>eff(( \bar{y}_{BM} ))</td>
<td>1.099414</td>
<td>1.334132</td>
<td>1.961767</td>
<td>5.262914</td>
</tr>
<tr>
<td>eff(( \bar{y}_{SD} ))</td>
<td>1.671444</td>
<td>2.100576</td>
<td>2.903828</td>
<td>4.920783</td>
</tr>
<tr>
<td>eff(( \bar{y}_{BM} ))</td>
<td>1.672733</td>
<td>2.102486</td>
<td>2.904315</td>
<td>4.903447</td>
</tr>
<tr>
<td>eff(( \bar{y}_{BM} ))</td>
<td>2.007205</td>
<td>2.264369</td>
<td>2.613332</td>
<td>3.112515</td>
</tr>
<tr>
<td>eff(( \bar{y}_{BM} ))</td>
<td>2.008323</td>
<td>2.263678</td>
<td>2.609383</td>
<td>3.102326</td>
</tr>
<tr>
<td>eff(( \bar{y}_{BM} ))</td>
<td>2.036949</td>
<td>2.273514</td>
<td>2.907244</td>
<td>6.244577</td>
</tr>
</tbody>
</table>
**Table-5: Efficiencies of the estimators for n=12, n’=4 and m=3.**

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{eff}({\bar{y}})$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{SD}})$</td>
<td>1.05258</td>
<td>1.333274</td>
<td>1.817967</td>
<td>2.856132</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{ST}})$</td>
<td>1.052898</td>
<td>1.333268</td>
<td>1.81568</td>
<td>2.841701</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{AO}})$</td>
<td>1.095674</td>
<td>1.231053</td>
<td>1.40461</td>
<td>1.635205</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{AOS}})$</td>
<td>1.095472</td>
<td>1.229776</td>
<td>1.401623</td>
<td>1.629365</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{BM}})$</td>
<td>1.098588</td>
<td>1.333279</td>
<td>1.962002</td>
<td>5.272846</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{SDSS}})$</td>
<td>1.888871</td>
<td>2.366586</td>
<td>3.288925</td>
<td>5.750091</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{JSS}})$</td>
<td>1.890671</td>
<td>2.369569</td>
<td>3.291312</td>
<td>5.733676</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{AOS}})$</td>
<td>2.412198</td>
<td>2.726525</td>
<td>3.164204</td>
<td>3.811702</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{OAOS}})$</td>
<td>2.414396</td>
<td>2.72665</td>
<td>3.160377</td>
<td>3.799801</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{BMS}})$</td>
<td>2.492892</td>
<td>2.72673</td>
<td>3.35366</td>
<td>6.657089</td>
</tr>
</tbody>
</table>

**Table-6: Efficiencies of the estimators for n=15, n’=5 and m=3.**

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{eff}({\bar{y}})$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{SD}})$</td>
<td>1.050763</td>
<td>1.330968</td>
<td>1.814932</td>
<td>2.852279</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{ST}})$</td>
<td>1.051083</td>
<td>1.330965</td>
<td>1.812657</td>
<td>2.837888</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{AO}})$</td>
<td>1.09457</td>
<td>1.229862</td>
<td>1.403409</td>
<td>1.634277</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{AOS}})$</td>
<td>1.094375</td>
<td>1.228594</td>
<td>1.400433</td>
<td>1.628449</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{BM}})$</td>
<td>1.097289</td>
<td>1.330973</td>
<td>1.958309</td>
<td>5.266478</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{SDSS}})$</td>
<td>2.087935</td>
<td>2.610542</td>
<td>3.641135</td>
<td>6.506117</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{JSS}})$</td>
<td>2.090183</td>
<td>2.614458</td>
<td>3.645175</td>
<td>6.490633</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{AOS}})$</td>
<td>2.784831</td>
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<td>3.672732</td>
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<tr>
<td>$\text{eff}(\bar{y}_{\text{OAOS}})$</td>
<td>2.788081</td>
<td>3.153052</td>
<td>3.669087</td>
<td>4.446733</td>
</tr>
<tr>
<td>$\text{eff}(\bar{y}_{\text{BMS}})$</td>
<td>2.921451</td>
<td>3.162086</td>
<td>3.803403</td>
<td>7.169375</td>
</tr>
</tbody>
</table>
6. CONCLUSION

The proposed ratio type estimator for population mean under SRS is always more efficient than the estimators defined by Sisodia and Dwivedi [4], Singh and Tailor [7] and Al-Omari et al [2]. The proposed estimator for population mean is more efficient under RSS design than under SRS design, for any value of \( \alpha \). Also, the proposed estimator is more efficient than the estimator defined by Jemain et al [1] and Al-Omari et al [2]. Also, at the optimum value of \( \alpha \), \( \bar{y}_{BMrss} \) is more efficient than \( \bar{y}_{BM} \). Hence, the use of proposed estimator of population mean will always be beneficial than the existing ones.

7. REFERENCES


